YEAR 11 SPECIALIST:
CIRCLE GEOMETRY

Angles in Circles

Syllabus points covered

1.3.6 an angle in a semicircle is a right angle

1.3.7 the size of the angle at the centre subtended by an arc of a circle is twice the size of the angle at the circumference subtended by the same arc

1.3.8 angles at the circumference of a circle subtended by the same arc are equal

1.3.9 the opposite angles of a cyclic quadrilateral are supplementary

We aim to…

- <u>Prove</u> that **angle at the centre is twice the angle at the circumference** and <u>use</u> this circle theorem to find unknown angles in circles

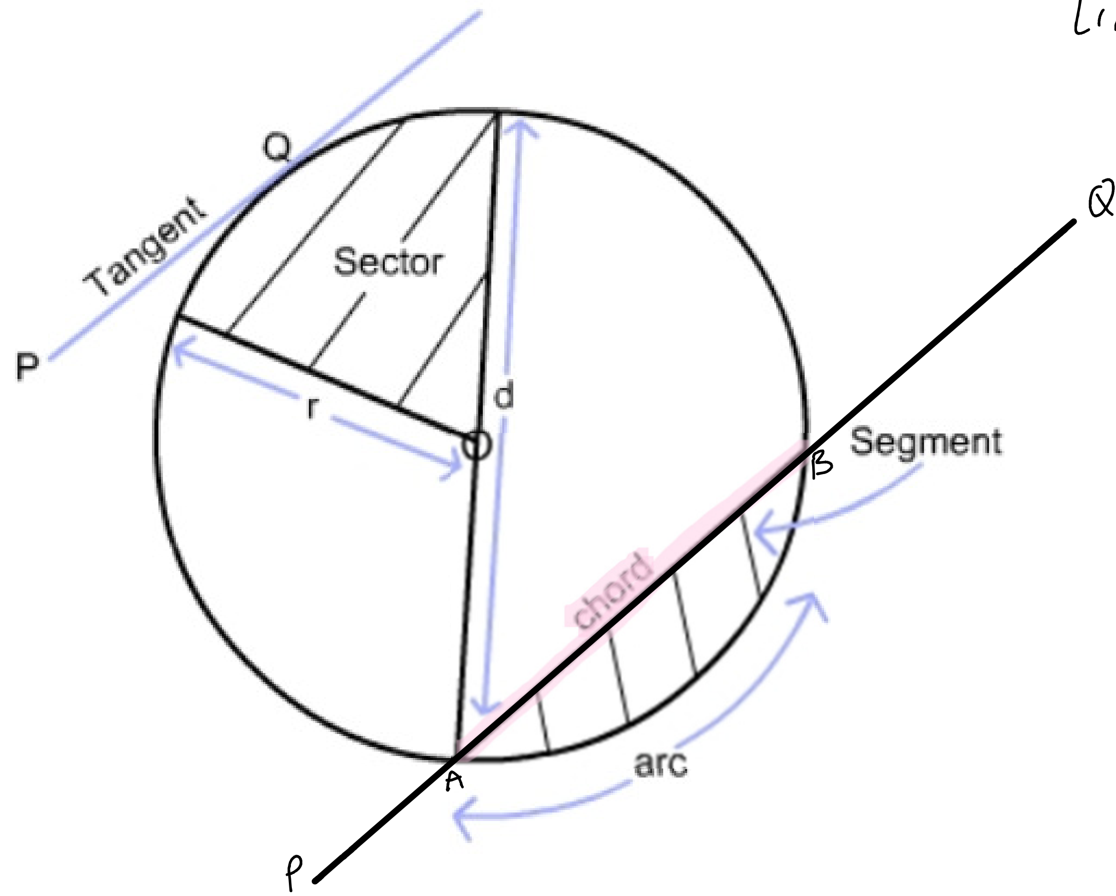
- <u>Prove</u> that **angle in a semicircle is a right angle** and <u>use</u> this circle theorem to find unknown angles in circles

- <u>Prove</u> that **angles at the circumference of a circle subtended by the same arc are equal** and <u>use</u> this circle theorem to find unknown angles in circles

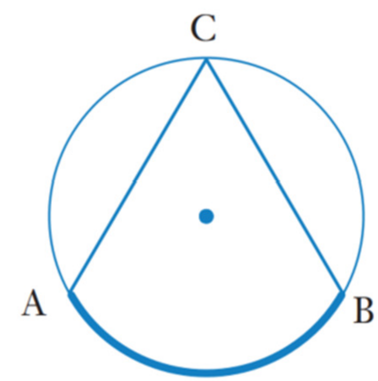
- <u>Prove</u> that **the opposite angles of a cyclic quadrilateral are supplementary** and <u>use</u> this circle theorem to find unknown angles in circles

Names in Circles

line AB is a chord
line PQ is a secant



Angles in circles



First though, note the use of the word *subtends*:

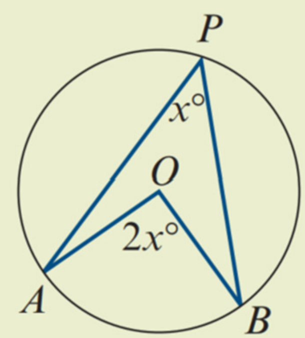
> We say that the arc AB shown in the diagram on the right **subtends** the angle ACB at C.

I.e. straight lines drawn from the extremities of the arc, to the point C, form the angle ACB.

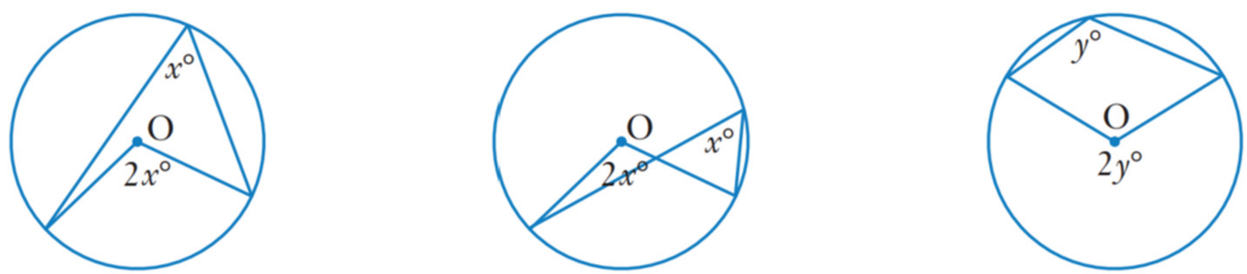
1.3.7 the size of the angle at the centre subtended by an arc of a circle is twice the size of the angle at the circumference subtended by the same arc

Theorem 1

The angle at the centre of a circle is twice the angle at the circumference subtended by the same arc.



Note: Situations involving this rule may not always look quite like the diagram above. See the diagrams below for examples.

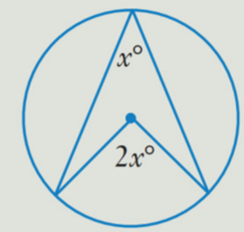


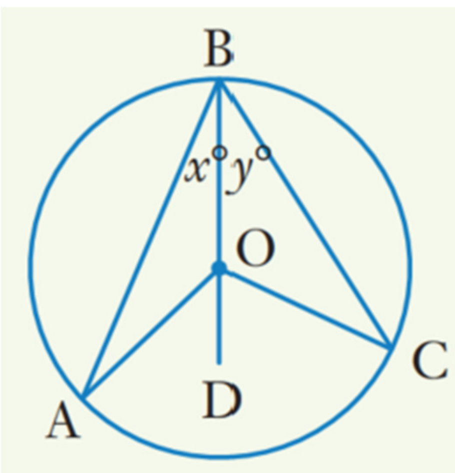
PROOF

The angle an arc subtends at the centre of a circle is twice the angle the same arc subtends at the circumference.

This result is referred to as:

Angle at the centre is twice the angle at the circumference.





$OA = OB$ (Radii)

$\therefore \triangle OAB$ is isosceles

thus $\angle OAB = \angle OBA = x°$ (Base angles of an isosceles triangle)

$\therefore \angle AOB = 180° - 2x°$ (Angle sum of a triangle)

and so $\angle AOD = 2x°$ (Angle of a straight line BD)

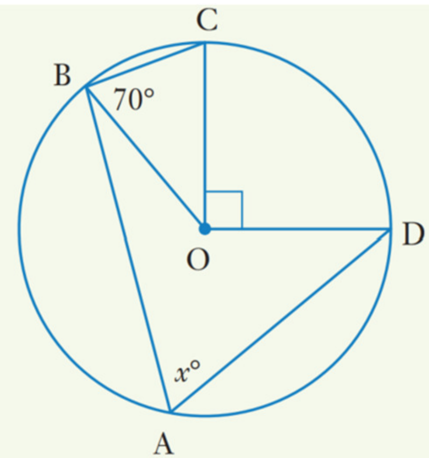
similar reasoning for $\triangle OCB$ gives $\angle COD = 2y°$

$$\begin{aligned}\angle AOC &= 2x° + 2y° \\ &= 2(x° + y°) \\ &= 2 \times \angle ABC\end{aligned}$$ as required.

EXAMPLE 4

In the diagram on the right, point O is the centre of the circle and points A, B, C and D lie on the circle.

$$\angle CBO = 70°,$$
$$\angle COD = 90°$$
and $$\angle BAD = x°.$$
Prove that $$x = 65.$$



EXAMPLE 4

In the diagram on the right, point O is the centre of the circle and points A, B, C and D lie on the circle.

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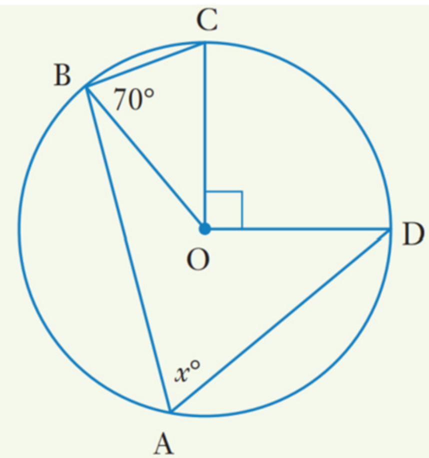


$OB = OC$ (Radii)

$\therefore \triangle OBC$ is isosceles

thus $\angle OCB = 70°$ (Base angles of an isosceles triangle)

and $\angle BOC = 40°$ (Angle sum of a triangle)

now $\angle BOD = \angle BOC + \angle COD$

$= 40° + 90°$

$= 130°$

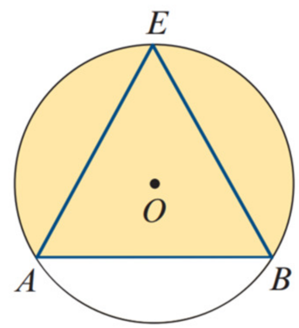
hence $\angle BAD = 65°$ (angle at the centre is twice angle at circumference)

thus $x = 65$ as required

1.3.8 angles at the circumference of a circle subtended by the same arc are equal

Angles in a segment

$\angle AEB$ is said to be an angle in segment AEB.



Theorem 2: Angles in the same segment

Angles in the same segment of a circle are equal.

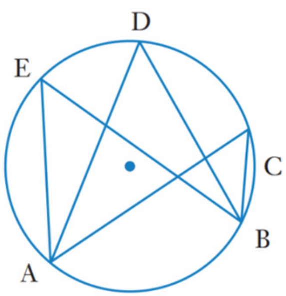


Diagram I

$\angle AEB = \angle ADB = \angle ACB$

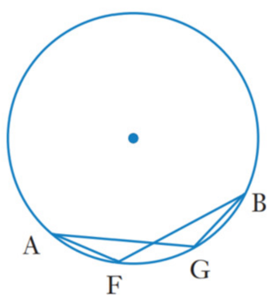


Diagram II

$\angle AFB = \angle AGB$

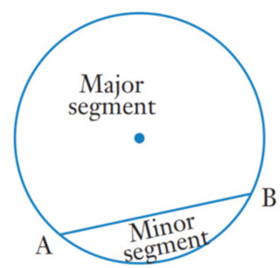
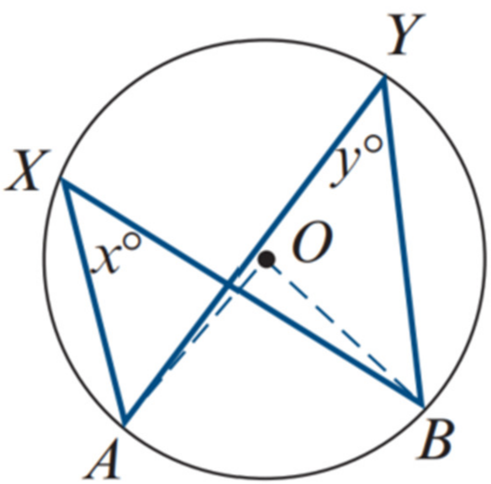
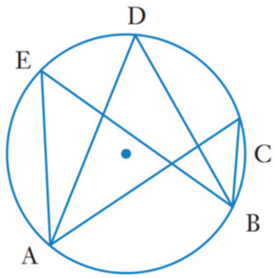


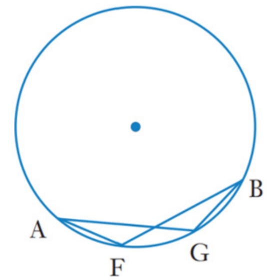
Diagram III

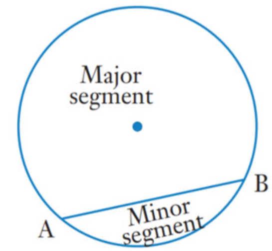
how would you prove this ?

Proof









Let $\angle AXB = x^\circ$ and $\angle AYB = y^\circ$

thus $\angle AOB = 2x^\circ = 2y^\circ$ (Theorem 1: angle at the centre is twice the angle at the circumference

$\therefore$ $x = y$ as required

1.3.6 an angle in a semicircle is a right angle

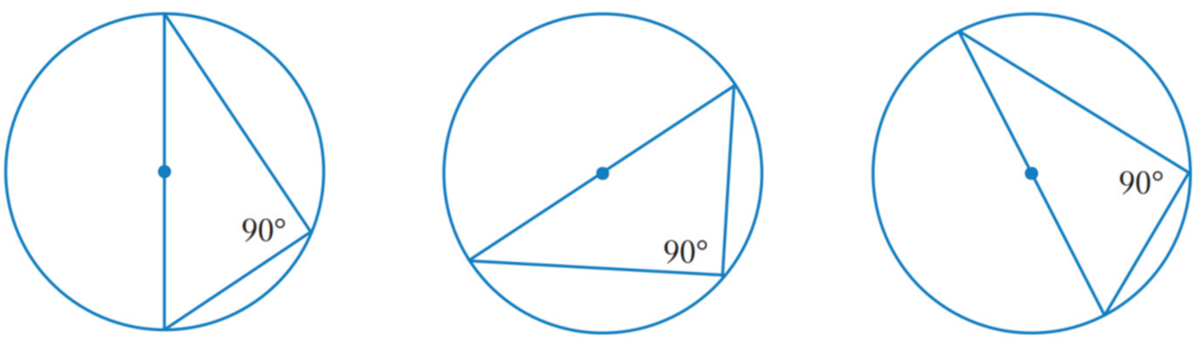
Theorem 3: Angle subtended by a diameter

The angle subtended by a diameter at the circumference is equal to a right angle (90°).

The angle a diameter subtends at the circumference is a right angle.

Or, as we tend to remember it:

Angles in a semicircle are right angles.



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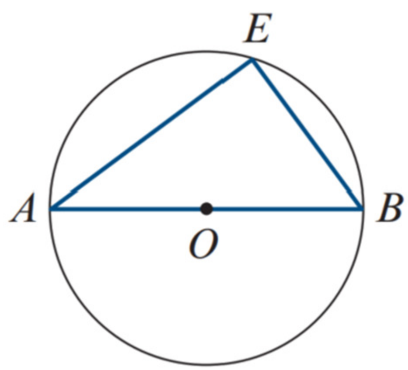


Proof

$\angle AOB = 2\angle AEB$ (Angle at the centre is twice the angle at the circumference)

but $\angle AOB = 180°$ (AB is a straight line)

$\therefore \angle AEB = 90°$ as required

1.3.9 the opposite angles of a cyclic quadrilateral are supplementary

Theorem 4

vertices lie on the circumference of a circle

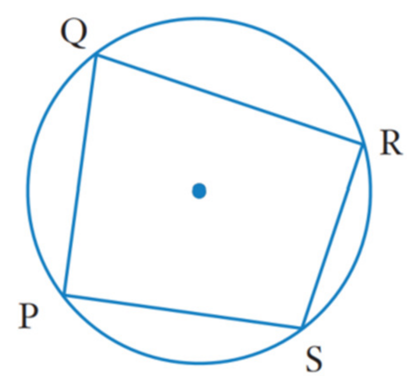
The opposite angles of a quadrilateral inscribed in a circle sum to 180°.

That is, the opposite angles of a cyclic quadrilateral are supplementary.

If the four vertices of a quadrilateral lie on the circumference of a circle we call the quadrilateral a **cyclic quadrilateral**.

Prove that:

The opposite angles of a cyclic quadrilateral add up to 180°.



Theorem 4

The opposite angles of a quadrilateral inscribed in a circle sum to 180°.

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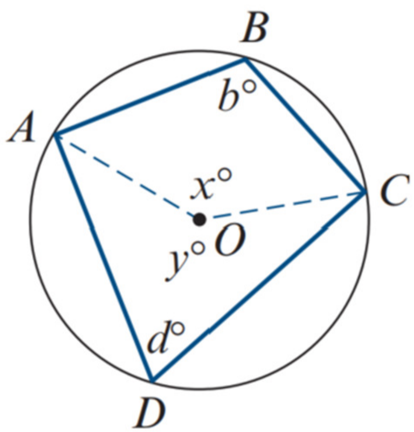

$x = 2d$ and $y = 2b$ (Angle at the centre is twice the angle at the circumference)

now $x + y = 360$ (angles at a point sum to 360°)

so $2b + 2d = 360$

hence $b + d = 180$ as required

Exercise to complete...

Cambridge Specialist Maths:

Exercise 8A